# UNITED STATES DEPARTMENT OF THE INTERIOR U.S. GEOLOGICAL SURVEY

# TIME SERIES ANALYSIS OF CENOZOIC ERA SEA LEVEL AND PALEOTEMPERATURE DATA

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Open File Report 83-566

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#### Abstract

A statistical analysis of Cenozoic era sea level and paleotemperature data was performed to determine the cycles of each data set and the correspondence between them. Accordingly, each of the four time series were first analyzed independently in the univariate mode of a spectral analysis. The two basic data sets were then analyzed in a paired cross-spectral analysis.

The prominent periodic cycles remaining in the data sets after linear trend removal, were: sea level surface from seismic stratigraphy—9.6 million years, updated version of sea level surface from seismic stratigraphy—9.5 million years, continental paleotemperatures from paleobotanical interpretations—9.6 million years, and marine paleotemperatures from foraminiferal isotopic data—12.7 million years. The cross—correlation properties between the data sets of continental paleotemperatures from paleobotanical interpretations and sea level surface from seismic stratigraphy at the common prominent period of 9.6 million years were: (1) The squared coherency value which measures cross correlation between the two data sets has the value 0.30, and (2) the amount by which the continental paleotemperatures from paleobotanical interpretations data lags the sea level surface from seismic stratigraphy data is 2.70 million years.

## 1. INTRODUCTION

# 1.1 Background

The statistical analysis of this problem was performed at the request of Harold Cousminer, Minerals Management Service, U.S. Department of the Interior, who presented the problem and furnished citations of the initial references. Additional references and sources of data were furnished by Lucy E. Edwards and Richard Z. Poore, Geologic Division, U.S. Geological Survey. The data being studied are fundamentally of two types: (1) Vail and Mitchum (1979) have charted the sea level surface from seismic stratigraphy for the period 3 Ma to 65 Ma (Ma = mega annum). Vail and others (1982) have updated the portion of these original data for the period 23 Ma to 65 Ma, and (2) Wolfe (1978) has graphed continental

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paleotemperatures from paleobotanical interpretations for the period 5 Ma to 62 Ma. Poore and Wolfe (1982) have updated the portion of these original data for the period 25 Ma to 62.5 Ma.

Rach of these authors has recognized the cyclical phenomena in their data over different time periods. Vail and Mitchum (1979, p. 469) indicate first-order cycles of 200 to 300 million years, second order cycles of 10 to 80 million years, and third-order cycles of 1 to 10 million years. Wolfe (1978, p. 702) recognizes a cycle of about 9.5 million years. Cousminer, in studying each of these two original data sets, realized that there appeared to be a correspondence between them, with cycles in the order of 9.6 to 10.6 million years. He requested a statistical analysis of these data sets, individually and together, to definitively determine the cycles of each data set and the correspondence between them.

# 1.2 Purpose and Scope

The purpose of this statistical analysis was to analyze the cyclic phenomena of each different determination of both sea level surface and paleotemperature, and to establish the periods of these cycles within each determination. The relationships of the cycles between the two data sets was also to be determined. Since there were two basic determinations, sea level surface and paleotemperatures, with an updated portion to each, it was decided to analyze each basic data set, and then each of the updated portions. Accordingly, each of the four time series was first analyzed independently in the univariate mode of a spectral analysis. Following this, the two basic data sets were analyzed in a paired cross-spectral analysis, as were the two updated data sets in a second, paired, cross-spectral analysis. The authors wish to thank Lucy E. Edwards for her constructive comments on the data analysis provided by her technical review of the manuscript.

## 2. SOURCES OF DATA

## 2.1 Sea Level Surface From Seismic Stratigraphy

The data used to determine this time series were obtained from Vail and Mitchum (1979) using their figure 3, the Cenozoic cycle chart. That chart represents a global cycle of relative change of sea level derived from seismic stratigraphic studies. The data source was a distribution copy of the paper which included the chart in reduced scale; and was obtained by Lucy Edwards direct from Vail. The reduced scale chart was enlarged in a precision photographic enlarger to a scale of 5 Ma per inch. This intermediate scale copy was further enlarged on a photolab copy camera to 2.5 Ma per inch. The graphical measurements were made on this final enlargement. Since only the cycles and not the absolute amplitude were of concern to this study, only relative height measurements were made. The useable data from this chart extend from 3 Ma to 65 Ma.

## 2.2 Updated Version of the Above Data Source

The data for this time series were obtained from an updated version of a portion of the Cenozoic cycle chart (Vail and others, 1982). This enlarged chart, already at a scale of 2.5 Ma per inch was also obtained by Lucy Edwards direct from Vail. The raw data obtained from this chart could not be correlated to the raw data obtained from the previous chart because of the updating changes to the sea level surface. Therefore, the data from this chart were considered to be a separate time series. For these data, the sea level surface was taken from the portion of the chart titled "Regional relative change in coastal onlap." The relative height measurements were made in the landward direction from the neat line of this portion of the chart. The useable data from this chart extend from 23 Ma to 65 Ma.

# 2.3 Continental Paleotemperatures from Paleobotanical Interpretations

The data used for this analysis were obtained from Wolfe (1978, figure 1, p. 695). The chart of this figure was enlarged from the journal article on a precision photographic enlarger to a scale of 5 Ma per inch, and then further enlarged on a photolab copy camera to a scale of 2.5 Ma per inch. The graphical measurements were made on this final enlargement. Since absolute amplitude is not of concern to this study, only relative height measurements were made.

There are two curves on the chart, which together cover the full 65 million years. The Pacific Northwest curve from 5 Ma to 50 Ma and the Mississippi Enbayment curve from 30 Ma to 62 Ma, with some missing data before 50 Ma. Therefore, the Pacific curve was used in its entirety and the Mississippi curve was used from 50 Ma to 62 Ma. The vertical height difference between the two curves at 50 Ma (200 units) was subtracted as a constant value from all of the Mississippi values, to fabricate a continuous curve.

## 2.4 Marine Paleotemperatures from Foraminiferal Isotopic Data

The data used for this part of the study were obtained from a chart by Poore and Wolfe (1982) and supplied by Poore. This chart is in part an updated version of figure 1 of the paper by Wolfe (1978). The furnished chart does not have mega annum designations, so these were added by Cousminer as described below. The furnished chart is at the scale of 5 Ma per inch. This chart was enlarged in a precision photographic enlarger to a scale of 2.5 Ma per inch. The data were measured to the marine paleotemperature curve in the climatic trends portion of this chart. The relative height measurements were made in the cold to warm ("C" to "W") direction from the neat line of this portion of the chart. The useable data from this chart extend from 25 Ma to 62.5 Ma.

The original chart was graduated into "Planktonic Foraminifera" zones and "Calcareous Nannofossil" zones. It is to these zones that Cousminer with the agreement of Poore, assigned the mega annum designations. Essentially, the Cenozoic cycle chart (figure 3) of Vail and Mitchum (1979) was used as a source of time data. The epoch and time boundary

lines from this chart of 25, 32, 35, 37, 40, 44, 49, 53.5 and 60 Ma were transferred to the marine surface chart. The marine surface chart line of 35 Ma was then changed to 37 Ma to correspond to the Vail and Mitchum (1979) chart. This renumbering of the time line from 35 to 37 Ma compressed the scale from 32 to 37 Ma, and expanded the scale from 37 to 40 Ma. The scale from 25 to 32, and from 40 to 65 remained at 2.5 Ma per inch.

#### 3. MEASUREMENT OF DATA

Each chart of the sea level surface or paleotemperatures was photographically enlarged to a scale of 2.5 Ma per inch. The fundamental graduations were subdivided into units of 0.5 Ma and relative height of the sea level surface or paleotemperature line was measured. The error in geologic time of the original measured data has been estimated by Cousminer to be in the order of 2.5 Ma. The data thus acquired from each chart formed an equally incremented time series of relative sea level height at 0.5 Ma increments.

## 4. STATISTICAL ANALYSIS OF DATA

The object of time-series analysis is to understand the generating causal mechanism. A typical time series may be composed of a trend, or long-term movement; oscillations about the trend, of greater or less regularity; a seasonal effect; and a random component (Kendall and Stuart, 1976, p. 263). The seasonal effect might be imposed by a cyclic phenomenon external to the main body of causal influences on the series, and as such might not always be present. Trend might be a smooth broad motion of the system over a long term, but it is always possible that a trend might be a part of a long, slow oscillation. When seasonal variation and trend have been removed, the remaining series might contain fluctuations of a more or less regular kind. This would represent the underlying structure of the data. The remaining nonregular deviations represent the random component.

## 4.1 Trend Removal

Removal of trend is often performed by long-term polynomial curve fitting, before the usual methods of time-series analysis are used. The data for this analysis were subjected to a linear fit using the General Linear Model (GLM) Procedure of the Statistical Analysis System (SAS) (Helwig and Council, 1979).

# 4.2 Spectral Analysis

The residuals of the linear fit were then analyzed to determine the underlying structure of the data. This was performed using the SAS--SPECTRA procedure (The SAS Institute, 1980). The SPECTRA procedure was

run with a simple five-point triangular smoothing weight. The data were centered using the adjusted mean (ADJMEAN) option of the SPECTRA procedure.

#### 4.2.1 The SPECTRA Procedure

In the spectral analysis procedure, the periodogram intensity values are computed from the coefficients of the sine and cosine waves of different amplitudes and wavelengths. The periodogram intensity value represents a sum-of-squares, in an analysis of variance sense, for the decomposition of the process into 2 degrees-of-freedom components for each of the m frequencies (The SAS Institute, 1980, p. 7.2). The smoothing weights required for the SPECTRA procedure are used to compute the smoothed spectral density values from the periodogram intensity values. The spectral density values and the periodogram intensity values are each computed relative to both the frequency and the period. Frequency and period are inversely related to each other. For purposes of this study, the periodogram intensity values on the basis of frequency are used for analyzing the data.

The SPECTRA procedure computes periods and frequencies as finite number functions of the data length. Thus, the first period is equal to the data length, the second as one-half of the data length, the third as one-third, etc.

#### 4.2.2 Prominent Periods

The prominent periods are determined from the periodogram intensity values plotted on a log scale against frequency. The prominent periods are indicated by those projections on the graph above a line determined by statistical analysis of the total set of intensity values. The prominent periods are determined as those period values which correspond to the frequency of the peaks of the prominent projections. Since the raw data set was measured in one-half mega-annum units, the period in full mega-annum units is one-half of the determined value.

A line is drawn on the graph to determine the prominent periods. This line represents the probability level of the random error component. In the data processing, the trend was removed by the linear fit, and the data were centered by the SPECTRA procedure. The periodogram intensity values represent sums-of-squares in the analysis of variance sense (The SAS Institute, 1980, p. 7.2). Since the values are plotted on the log scale, the geometric mean (GM) represents the arithmetic mean in the log scale. The standard deviation about the geometric mean (<sup>S</sup>GM) then represents the random error component in the data.

The lines which are drawn on the graph represent the measure of total error remaining in the data, based upon the geometric mean value and the values of both one and two standard deviations about the geometric mean, computed in the manner:

based upon one standard deviation:

$$ST = \left[ (GM)^2 + (S_{GM})^2 \right]^{-1/2}$$

based on two standard deviations:

$$ST(2) = \left[ (GM)^2 + (2 S_{GM})^2 \right]^{-1/2}$$

The statistical validity of the selection of prominent periods is determined by application of the concept of one-sided, distribution-free tolerance limits (Natrella, 1963, p. 2-15, and table A-31).

## 4.3 Cross-Spectral Analysis

Cross-spectral analysis is performed using the "cross" option of the SPECTRA procedure (The SAS Institute, 1980) only on the basis of a paired comparison—two time series at a time. More than two series can be analyzed in a run, and all combinations of pairs are utilized; all series must therefore have the same time span. Output of the procedure includes the amplitudes, the squared coherencies, and the phase differences of the cross spectra.

The squared coherency spectrum provides a nondimensional measure of the correlation between the two time series. The phase-difference spectrum indicates the amount by which the components of frequency 1 lag or lead those of frequency 2. Thus, the cross correlation properties of two time series can be described by the squared-coherency spectrum and the phase-difference spectrum (Jenkins and Watts, 1968, p. 359).

Because the cross-spectral analysis is wanted after the trend removal, a linear least-squares fit is made to each data set being entered. The cross-spectral procedure is then applied to the residuals of this fit. For this analysis, the procedure is restricted to computing the squared coherency value (K) and the phase-difference value (PH). The periods are computed as harmonics of the length of the data set input. This will be the shorter of the two original data sets.

# 5. RESULTS OF DATA ANALYSIS 5.1 Trend Removal

It is recalled that trend was removed with a linear fit. The statistical results of this linear adjustment are given in table 1 for each of the data sets used.

Table 1.--Statistical results of linear fit to remove trends

D	egree of freedom			Mean square
Data Set	for error	Model F	$\frac{R}{2}$	for error
Vail and Mitchum, 19	79 123	141.94	0.5357	3291.78
Vail and Others, 198	2 83	58.36	0.4128	477.36
Wolfe, 1978	113	96.94	0.4617	8622.08
Poore and Wolfe, 198	2 75	37.06	0.3307	480.94

In this table, the model F "tests how well the model as a whole (after adjusting for the mean) accounts for the dependent variable's behavior" (Helwig, 1978, p. 61). The coefficient of determination,  $R^2$ , measures how much variation in the dependent variable can be accounted for by the model (how well the model fits the data). The mean square for error is an estimate of the variance of the residuals.

Since the linear fit is only for trend removal, not for model fit, it is not expected that the model will fit the data. This is evidenced from the low value of  $\mathbb{R}^2$ , 0.5 and lower, and from the relatively large values for the mean square for error. It is expected that the residuals will now contain the underlying structure of the time series plus the random component.

# 5.2. Spectral Analysis

The prominent periods are those designated as having peaks on the frequency plot which project significantly above a statistically determined line. This line represents the total error remaining in the data after removal of the trend and the cyclical phenomena. The values for the line are computed from the geometric mean and standard deviation of the periodogram intensity values for that set of data. The computed values for the geometric mean, the standard deviation, and the total error based on one and two standard deviations as given in table 2.

Table 2.--Computed values of the geometric mean, standard deviation, and total error based on one and two standard deviations for each of the four data sets

data set	GM	_sgm_	s	<sup>8</sup> T(2)	n
Vail and Mitchum (1979)	1461	17,259	17,320	34,548	58
Vail and Others (1982)	290	2,069	2,089	4,148	39
Wolfe (1978)	249	66,170	66,171	147,961	58
Poore and Wolfe (1982)	127	2,370	2,374	4,742	39

GM = geometric mean.

It is noted in several cases that the line based on one standard deviation is very close to several peaks in the graphs. The Cenozoic time values for the original measured data are too much in error (2.5 Ma) for having much confidence in so close a tolerance. Accordingly, a total error line based on two standard deviations was used for the final selection.

SGM = standard deviation.

ST = total error based on one standard deviation.

 $<sup>^{</sup>S}T(2)$  = total error based on two standard deviations.

n = number of periodogram intensity values.

The measured data from all four sources covered time domains as follows:

- (1) Vail and Mitchum (1979), 3 Ma to 65 Ma,
- (2) Vail and others (1982), 23 Ma to 65 Ma,
- (3) Wolfe (1978), 5 Ma to 62 Ma,
- (4) Poore and Wolfe (1982), 25 Ma to 62.5 Ma.

However, the periods obtained from the spectral analysis are ultimately to be compared to one of the other sources in the following manner:

- (1) Vail and Mitchum (1979) to Wolfe (1978),
- (2) Vail and others (1982) to Poore and Wolfe (1982).

Accordingly, the larger time domain of each pair was shortened to coincide with that of the other. The final time domains are as follows:

- (1) Vail and Mitchum (1979), 5 Ma to 62 Ma,
- (2) Vail and others (1982), 25 Ma to 62.5 Ma,
- (3) Wolfe (1978), 5 Ma to 62 Ma,
- (4) Poore and Wolfe (1982), 25 Ma to 62.5 Ma.

Appendix A contains plots which show for each data set:

- (1) a plot of the relative sea level height or paleotemperature measurements over age in 0.5 Ma units.
- (2) a plot of the residuals after trend removal over age in 0.5 Ma units.
- (3) a plot of the periodogram intensity values (PO1) over frequency.

Comparison of the plot of the measurements with the plot of the residuals, for each data set, indicates the following results concerning the removal of long-term trend:

- (1) Vail and Mitchum (1978). There is a definite long term trend in the series data, which was probably removed.
- (2) Vail and others (1982). There is also a definite linear trend in the series data, which probably still remains in the data.
- (3) Wolfe (1978). There is a definite linear trend in the series data, which was probably removed.
- (4) Poore and Wolfe (1982). There may be a linear trend in the series data, but a reverse trend may have been introduced.

Inspection of the plots of the periodogram intensity values over frequency, for each data set, indicates the following results:

(1) Vail and Mitchum (1979). It is noted that the prominent periods occur at approximately 53.6 (determined by interpolation) and at 19.2 in 0.5 Ma units and thus at 26.8 Ma and at 9.6 Ma. Both of these periodogram intensity values exceed the total error value based on two standard deviations of 34,548. Based on a sample size of 58 periodogram intensity values, there is at least 95-percent confidence that 90 percent of the population lies below these two prominent periods. These periods are listed in table 3.

Table 3.—Summary of time domain and prominent periodic cycles after linear trend removal

	Prominent Periods		
Data	<b>@26.8</b>	12.7	9.5/9.6
Vail and Mitchum (1979)	<b>@26.8</b>		9.6
Vail and other (1982)			9.5
Wolfe (1978)			9.6
Poore and Wolfe (1982)		12.7	

- (2) Vail and others (1982). It is noted that the prominent period occurs at 19.0 in 0.5 units and thus at 9.5 Ma. This periodogram intensity value exceeds the total error value based on two standard deviations of 4,148. Based on a sample size of 39 periodogram intensity values, there is at least 90-percent confidence that 90 percent of the population lies below this prominent period. This period is listed in table 3.
- (3) Wolfe (1978). It is noted that the prominent period occurs at 19.2 in 0.5 Ma units and thus at 9.6 Ma. This periodogram intensity value exceeds the total error based on two standard deviations of 147,961. Based on a sample size of 58 periodogram intensity values, there is at least 95-percent confidence that 95 percent of the population lies below this prominent period. This period is listed in table 3.
- (4) Poore and Wolfe (1982). It is noted that the prominent period occurs at 25.3 in 0.5 Ma units and thus at 12.7 Ma. This periodogram intensity value exceeds the total error value based on two standard deviations of 4,742. Based on a sample size of 39 periodogram intensity values, there is at least 90-percent confidence that 90 percent of the population lies below this prominent period. This period is listed in table 3.

Appendix B contains tables which show for each data set:

- (1) a listing of the age by 0.5 Ma units, the relative sea level height or paleotemperature measurements, and the residuals after trend removal.
- (2) a listing of the results of the spectral analysis showing the underlying structure of the data. This structure is tabulated by frequency, period, the periodogram intensity value (PO1), and the smoothed-spectral density (SO1).

### 5.3 Cross-Spectral Analysis

## 5.3.1 Sea Level Surface Versus Continental Paleotemperatures

This cross-spectral analysis compares the cross correlation properties of the sea level surface data from seismic stratigraphy (Vail and Mitchum, 1979) and the continental paleotemperatures from

paleobotanical interpretations (Wolfe, 1978). These two series cover essentially the same long-term time interval: Vail and Mitchum from 3 Ma to 65 Ma; and Wolfe from 5 Ma to 62 Ma. Because the cross-spectral analysis requires all series to have the same time span, the records from Vail and Mitchum were used from 5 Ma to 62 Ma only. Accordingly, the output periods were computed as harmonics of the Wolfe (1978) data set.

The two data sets were input in the sequence: Wolfe (1978) as data set "01," and Vail and Mitchum (1979) as data set "02." The resulting cross-correlation properties are given in table 4. There is only one

Table 4.--Cross-correlation properties between data of Wolfe (1978) and Vail and Mitchum (1979)

				РН0102
Period (Ma)	Period (t Ma)	<u>_K</u>	Radians	Lag (Ma)
9.6	19.2	0.30	-1.7683	-2.70

K = squared-coherency values between series 1 and series 2.

PH = phase-difference values by which series 1 lags series 2.

01 = series 1: Wolfe (1978) data.

02 = series 2: Vail and Mitchum (1979) data.

Period (Ma) represents the periods for which prominent periodogram intensity values were obtained in the univariate spectral analysis for data set "01."

Period ( $\frac{1}{2}$  Ma): values are in the units of the SPECTRA output. lag (Ma): values in Ma by which series 1 lags series 2.

prominent period common to both data sets; this is at 9.6 Ma. The squared coherency value (K), which measures cross correlation at the prominent period between the two data sets has the value 0.30. The amount by which the Wolfe (1978) data lags the Vail and Mitchum (1979) data is 2.70 Ma.

## 5.3.2 Update of Sea Level Surface Versus Marine Paleotemperatures

This cross-spectral analysis compares the cross correlation properties of the updated version of the sea level surface from seismic stratigraphy (Vail and others, 1982) and the marine paleotemperatures from foraminiferal isotopic data (Poore and Wolfe, 1982). These two series cover essentially the same shorter term time interval: Vail and others from 23 Ma to 65 Ma; and Poore and Wolfe from 25 Ma to 62.5 Ma. Because the cross-spectral analysis requires all series to have the same time span, the records of Vail and others were used from 25 Ma to 62.5 Ma and the output periods were computed as harmonics of the Poore and Wolfe (1982) data set. The two data sets were input in the sequence Poore and Wolfe (1982) as data set "01", and Vail and others (1982) as data set "02," with no prominent periods common to both data sets.

# 6. DISCUSSION 6.1 Spectral Analysis

The values in table 3 are the basis for discussion of the results of the univariate spectral analysis for each data set. It is noted that three of the four data sets have a prominent period at 9.5/9.6 Ma in common. This period is common to the Vail and Mitchum (1979), Vail and other (1982), and Wolfe (1978) data sets. The Poore and Wolfe (1982) data set has no prominent period in common with the other three data sets. It is noted that the data set of Vail and others (1982) which is an update of the earlier Vail and Mitchum (1979) data set, has eliminated the longer period. It is, of course, possible that if the data set of Vail and others (1982) had been longer, that the longer period (12.7 Ma) might still be determinable. It is also noted that the data set of Poore and Wolfe (1982) which covers the earlier portion of the Wolfe (1978) data set, and is determined from a different type of observation, has eliminated the 9.6 Ma period and has instead a period of 12.7 Ma. It remains, however, to determine to what extent the cycles from sea level surface and paleotemperature agree with each other. This will be shown from the cross-spectral analysis.

# 6.2 Cross-Spectral Analysis

The point of departure for the discussion of the cross-spectral analysis are the values in table 4. There is only one common prominent period between the Vail and Mitchum (1979) data set and the Wolfe (1978) data set; this is at 9.6 Ma. The data in table 4 indicate that, over the entire Cenozoic Era, and at the prominent period of 9.6 Ma, the paleotemperatures are not well correlated (0.30) with the sea level surface values, and that the period of the paleotemperatures (Wolfe, 1978) lag those of the sea level surface (Vail and Mitchum, 1979) by 2.7 Ma. There is no common prominent periods between the Vail and others (1982) data set and the Poore and Wolfe (1982) data set, so a cross-spectral analysis is not available.

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APPENDIX A -- PLOTS OF TIME SERIES DATA

Figure Al. -- Plot of sea level (in arbitrary units) over age (in mega annum units) data from Vail and Mitchum, 1979.

61.0

57.5

54.0

50.5

47.0

43.5

40.0

36.5

33.0 AGE

29.5

26.0

22.5

19.0

12.0

8.5

5.0

125

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275

250

225

200

175

150

SEALVL |

400

350

325

300

375

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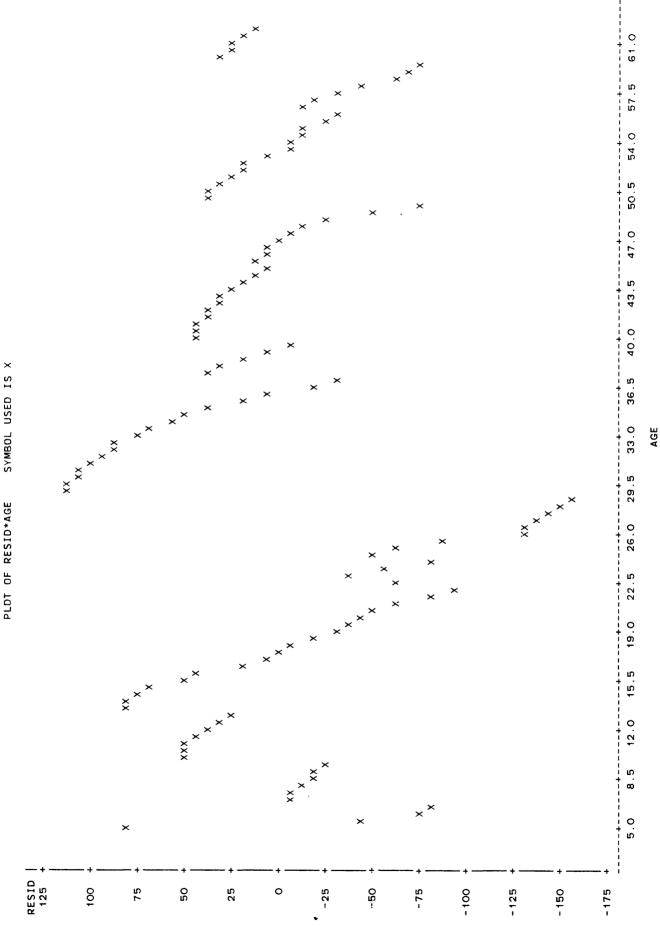


Figure A2.--Plot of residuals (in arbitrary units) over age (in mega annum units) data from Vail and Mitchum, 1979.

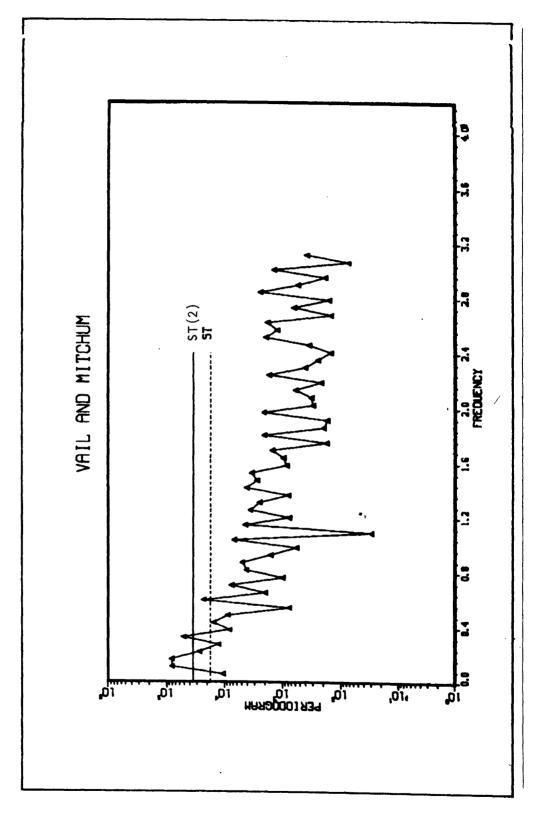
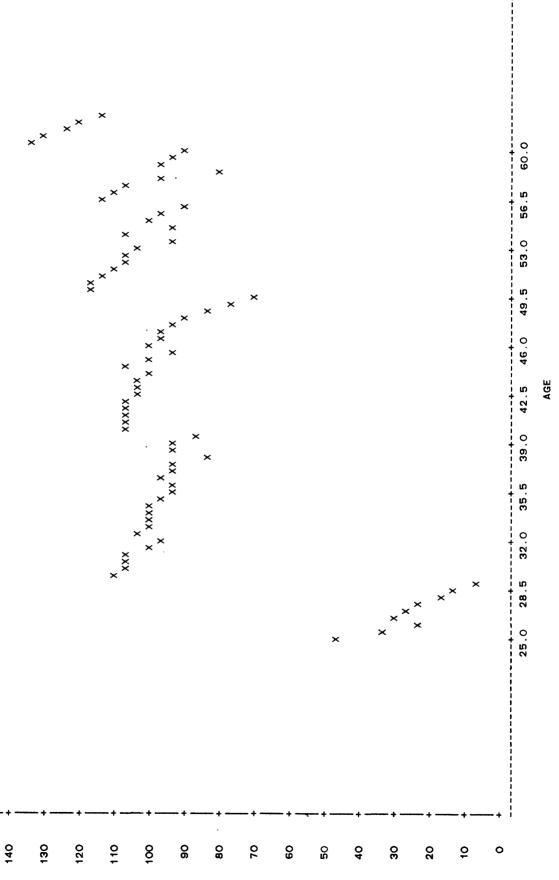


Figure A3. --Plot of periodogram intensity value (P\_01, in arbitrary units) over frequency (in cycles per one-half mega annum units), with indicated period values (in mega annum units) data from Vail and Mitchum, 1979.

27

SEALVL |

150



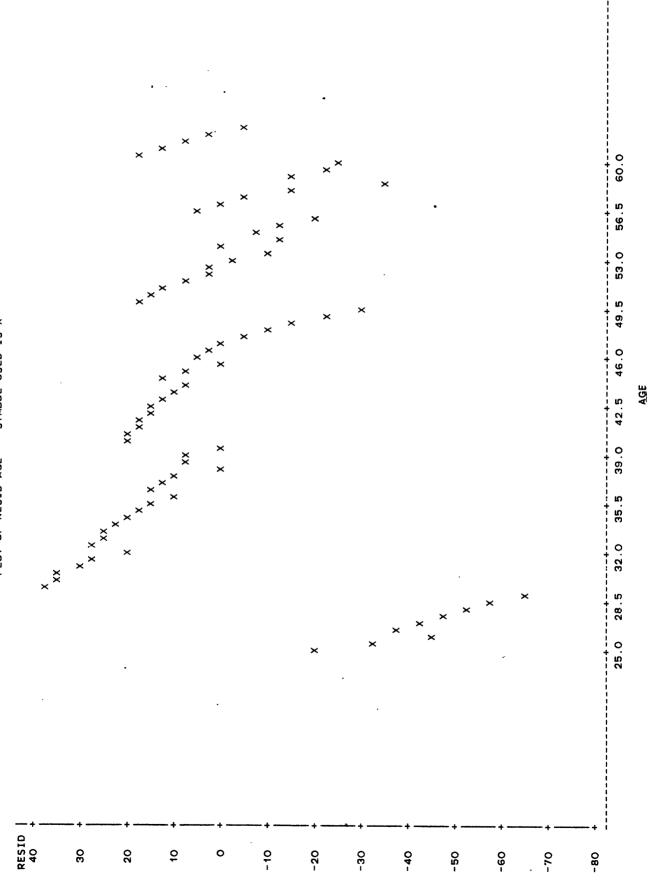


Figure A5. --Plot of residuals (in arbitrary units) over age (in mega annum units) data from Vail and Others 1982.

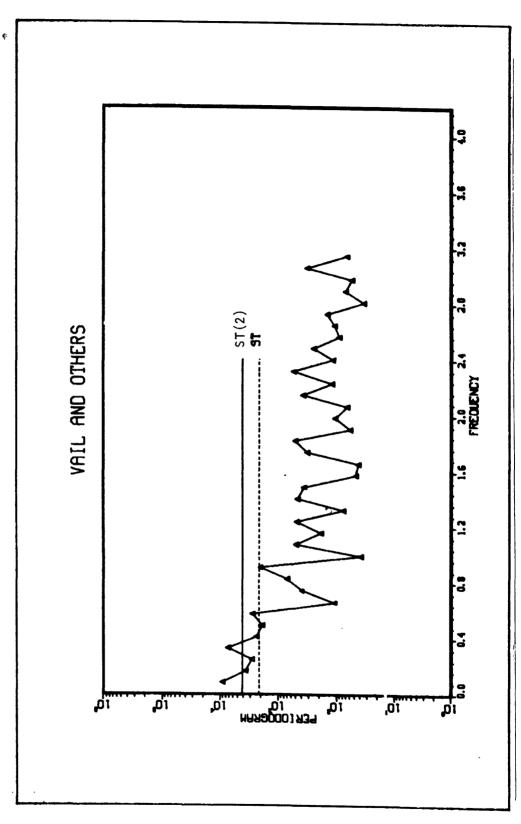


Figure A6.——Plot of periodogram intersity value (P\_O1, in arbitrary units) over frequency (in cycles per one-half mega annum units), with indicated period value (in mega annum units) data from Vail and Others, 1982.

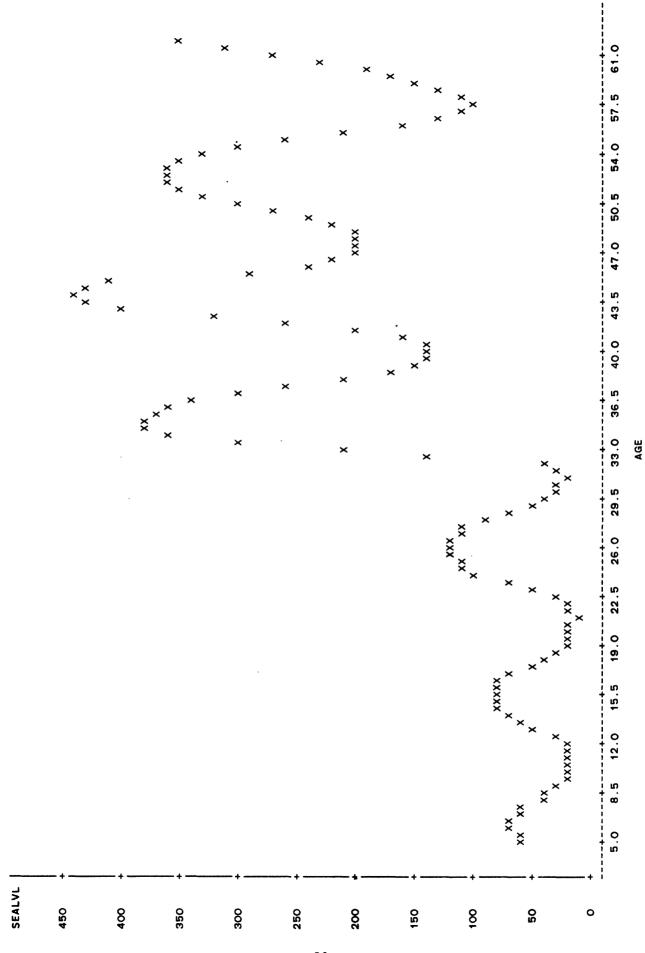


Figure A7.---Plot of paleotemperature (in arbitrary units) over age (in mega annum units) data from Wolfe, 1978.

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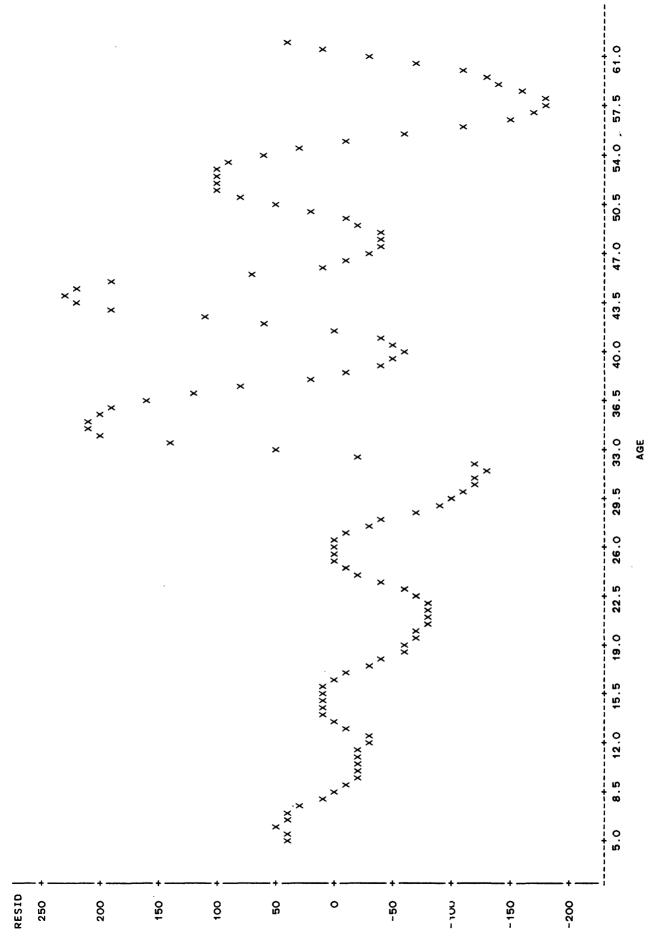


Figure A8. -- Plot of residuals (in arbitrary units) over age (in mega annum units) data from Wolfe, 1978.

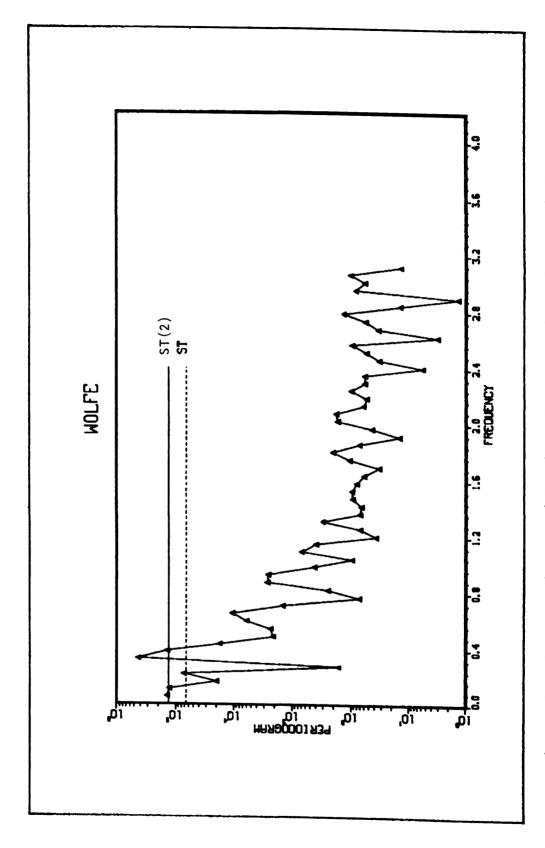


Figure A9.--Plot of periodogram intensity value (P\_01, in arbitrary units) over frequency (in cycles per one-half mega annum units), with indicated period values (in mega annum units) data from Wolfe, 1978.

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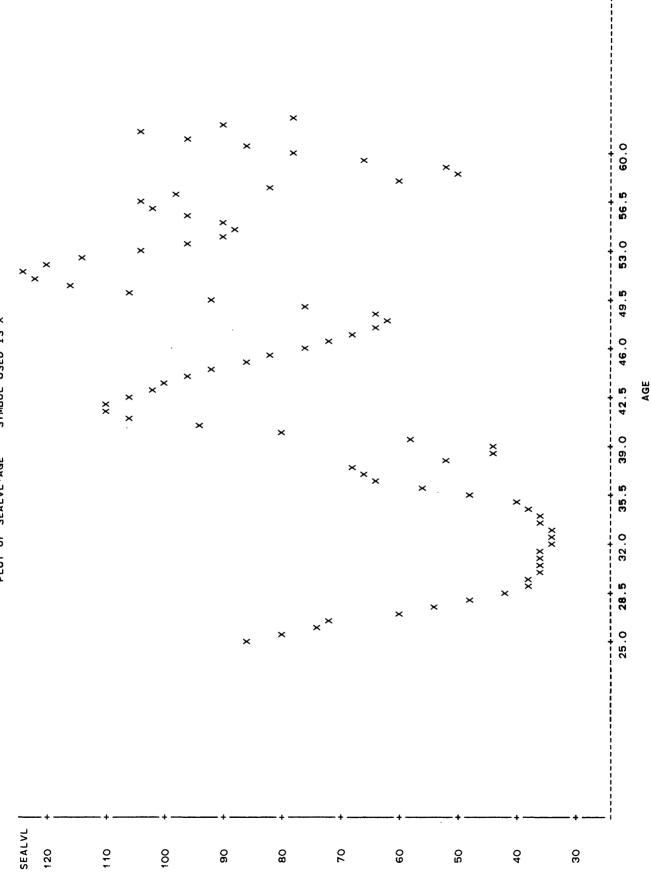


Figure A10. -- Plot of palectemperature (in arbitrary units) over age (in mega annum units) data from Poore and Wolfe, 1982.

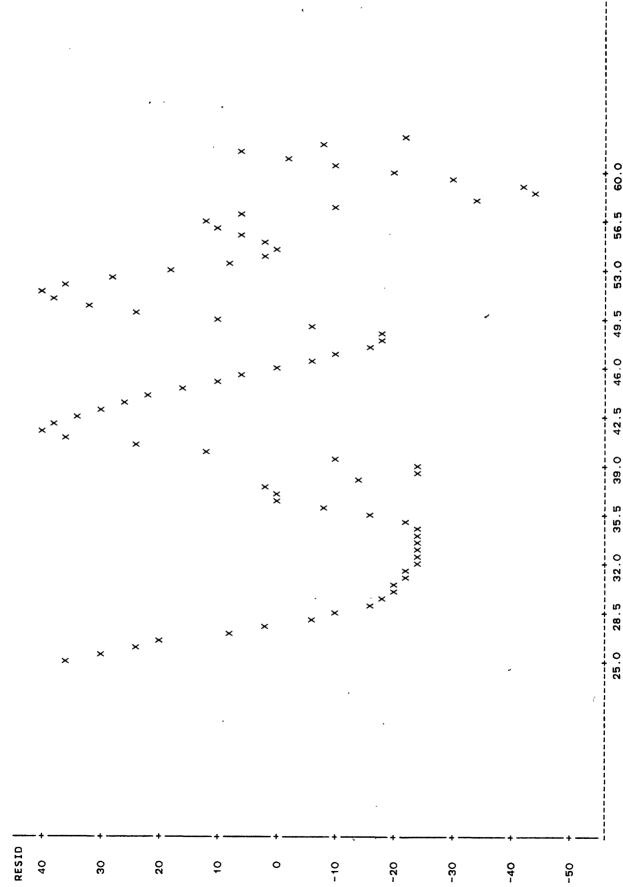


Figure All. -- Plot of residuals (in arbitrary units) over age (in mega annum units) data from Poore and Wolfe, 1982.

AGE

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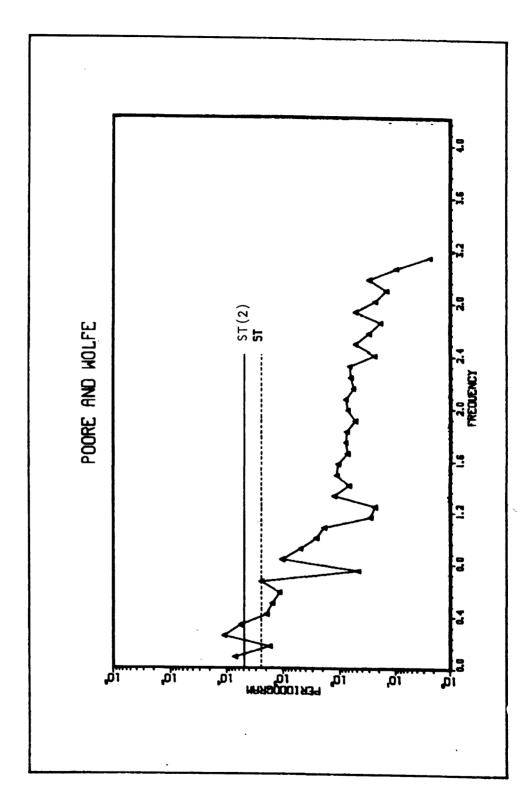


Figure A12.--Plot of periodogram intensity value (P\_01, in arbitrary units) over frequency (in cycles per one-half mega annum units), with indicated period values (in mega annum units) data from Poore and Wolfe, 1982.

APPENDIX B -- TIME SERIES DATA

Table B1.--Age (in mega annum units), sea level, and residuals (in arbitrary units) from Vail and Mitchum, 1979.

MATI	4 5 10	MITCHUM	4070	DATA
VAII	A NH 1	MID I (')—II JMI	1979	אוארו

OBS	AGE	SEALVL	RESID
12345678901123456789012345678901234567890123456789012345678901234567890123456	5.5.0.5.0.5.0.5.0.5.0.5.0.5.0.5.0.5.0.5	280 159 132 124 2003 202 198 190 271 271 265 265 265 265 265 265 265 265 265 265	78.60 -44.18 -72.96 -82.74 -4.52 -7.30 -10.08 -15.86 -21.64 -27.42 51.80 50.02 48.46 38.68 31.90 22.12 83.34 81.56 68.00 50.22 44.44 16.66 76.78 68.00 -5.68 -16.46 -29.24 -35.02 -43.80 -56.26 -78.14 -93.92 -64.70 -40.48 -56.66 -79.04 -51.82 -64.60 -84.38 -129.16 -132.94 -137.72 -143.50 -148.28 -156.06 111.16 109.38 107.60 104.82 101.04 96.26 90.48 84.70

Table B1. -- continued

VAIL AND MI	TCHUM.	1979	DATA
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OBS	AGE	SEALVL	RESID
57 . 58	33.0 33.5	378 371	76.919 68.139
59	34.0	364	59.359
60	34.5	355	48.579
61	35.0	345	36.799
62	35.5	331	21.019
63	36.0	315	3.239
64	36.5	294	-19.541
65	37.0	283	-32.321
66	37.5	355	37.899
67 68	38.0 38.5	349 339	30.119 18.339
69	39.0	326	3.559
70	39.5	319	-5.221
71	40.0	371	44.999
72	40.5	371	43.219
73	41.0	371	41.439
74	41.5	370	38.659
75	42.0	369	35.879
76 77	42.5 43.0	368 366	33.099
78	43.5 43. <b>5</b>	363	29.319 24.539
79	44.0	359	18.759
80	44.5	354	11.979
81	45.0	347	3.199
82	45.5	355	9.419
83	46.0	355	7.639
84	46.5	354	4.859
85	47.0	352	1.079
86 87	47.5 48.0	349 344	-3.701 -10.481
88	48.5	333	-23.261
89	49.0	307	-51.041
90	49.5	285	-74.821
91	50.0	400	38.399
92	50.5	399	35.619
93	51.0	396	30.839
94 95	51.5 52.0	393	26.059
96	52.5	385 387	16.279 16.499
97	53.0	380	7.719
98	53.5	368	-6.061
99	54.0	368	-7.841
100	54.5	366	-11.621
101	55.0	364	-15.401
102	55.5	358	-23.181
103 104	56.0 56.5	350 371	-32.961 -13.741
105	57.0	366	-20.521
106	57. <b>5</b>	358	-30.301
107	58.0	346	-44.081
108	58.5	328	-63.861
109	59.0	322	-71.641
110	59.5	318	-77.421
111	60.0	426	28.799
112	60.5	425	26.019

Table B1. -- continued

# VAIL AND MITCHUM, 1979 DATA

OBS	AGE	SEALVL	RESID
113	61.0	423	22.2389
114	61.5	421	18.4589
115	62.0	418	13.6789

Table B2.—Frequency (in cycles per one-half mega annum units), period (in one-half annum units), periodogram intensity value (P\_O1), and spectral density estimate (S\_O1) from Vail and Mitchum (1979, p. 11 and 12).

<b>VAIL</b>	AND MITCHUM,	1979 DATA	WITH WEIGHTS	1 2 3 2
OBS	FREQ	PERIOD	P_01	S_01
1	.0.0000		0.0	2072.53
2	0.05464	115.000	10584.9	2691.35
3	0.10927	57.500	80151.6	4069.17
4	0.16391	38.333	80571.7	4235.41
5	0.21855	28.750	26859.4	3524.14
6	0.27318	23.000	12691.6	2504.43
7	0.32782	19.167	51314.6	2105.57
8	0.38245	16.429	8249.5	1589.29
9	0.43709	14.375	15448.3	1171.52
10 11	0.49173 0.54636	12.778 11.500	8778.8 779.4	793.77 731.88
12	0.60100	10.455	22731.7	794.70
13	0.65564	9.583	1966.4	600.92
14	0.71027	8.846	7413.2	486.66
15	0.76491	8.214	994.0	291.09
16	0.81955	7.667	4147.6	293.11
17	0.87418	7.188	4851.1	244.21
18	0.92882	6.765	1603.4	235.40
19	0.98346	6.389	570. <b>6</b>	207.29
20	1.03809	6.053	6822.3	244.72
21	1.09273	5.750	29.5	211.25
22	1.14736	5.476	4406.8	222.72
23	1.20200	5.227	774.4	183.38
24 25	1.25664 1.31127	5.000 4.792	3538.8 2496.3	197.72
26	1.36591	4.792	796.7	186.35 193.59
27	1.42055	4.423	4141.2	223.29
28	1.47518	4.259	2689.9	218.55
29	1.52982	4.107	3359.7	196.89
30	1.58446	3.966	849.5	135.88
31	1.63909	3.833	968.2	98.03
32	1.69373	3.710	1473.2	84.76
33	1.74836	3.594	176.7	76.88
34	1.80300	3.485	2027.5	74.87
35	1.85764	3.382	195.1	63.95
36	1.91227	3.286	168.4	65.29
37 38	1.96691 2.02155	3.194 3.108	2079.4 302.3	68.04 57.19
39	2.07618	3.026	320.6	44.67
40	2.13082	2.949	592.8	42.74
41	2.18546	2.875	220.8	52.29
42	2.24009	2.805	1670.5	62.82
43	2.29473	2.738	404.4	47.96
44	2.34936	2.674	250.3	34.31
45	2.40400	2.614	148.9	35.35
46	2.45864	2.556	351.9	59.46
47	2.51327	2.500	1943.0	96.83
48	2.56791	2.447	1234.6	103.35
49 50	2.62255 2.67718	2.396	1800.2	94.97
51	2.73182	2.347 2.300	147.1 633.4	59.28 59.43
52	2.78646	2.255	160.9	64.15
53	2.84109	2.212	2404.9	83.58
54	2.89573	2.170	548.8	74.18

2.130

2.091

185.3

1401.9

61.33

50.02

55

56

2.95037

3.00500

Table B2. -- continued

VATI	AND MITCHIM	1979 DATA	WITH WEIGHTS	1 2 3	2 1

OBS	FREQ	PERIOD	P_01	S_01
57	3.05964	2.05357	73.622	35.8350
58	3.11427	2.01754	384.607	37.5976

Table B3.--Age (in mega annum units), sea level, and residuals (in arbitrary units) data from Vail and Others, 1982.

	'VAIL AND	OTHERS,	1982
OBS	AGE	SEALVL	RESID
1	25.0	46.5	-18.893
2	25.5	34.0	-32.101
3	26.0	22.5	-44.309
4	26.5	30.0	-37.516
5	27.0	25.5	-42.724
6 7	27.5	22.0	-46.932 -52.140
8	28.0 28.5	17.5 13.5	-52.140 -56.847
9	29.0	5.0	-66.055
10	29.5	110.0	38.237
11	30.0	108.0	35.529
12	30.5	107.0	33.822
13	31.0	105.0	31.114
14	31.5	101.5	26.906
15 16	32.0	96.0	20.698
17	<b>32.5</b> 3 <b>3</b> .0	102.5 101.5	26.490 24.783
18	33.5	101.5	24.075
19	34.0	101.5	23.367
20	34.5	100.0	21.159
21	35.0	97.5	17.952
22	35.5	95.0	14.744
23 24	36.0	92.0	11.036 15.328
25	36.5 37.0	97.0 95.0	12.621
26	37.5	92.0	8.913
27	38.0	85.0	1.205
28	38.5	93.0	8.497
29	39.0	92.0	6.790
30	39.5	85.5	-0.418
31 32	40.0 40.5	107.0 106.5	20.374 19.166
33	41.0	106.5	18.458
34	41.5	106.5	17.751
35	42.0	105.0	15.543
36	42.5	104.0	13.835
37	43.0	104.0	13.127
38	43.5	102.0	10.420
39 40	44.0 44.5	100.0 105.0	7.712 12.004
41	45.0	100.0	6.296
42	45.5	94.0	-0.411
43	46.0	100.5	5.381
44	46.5	98.0	2.173
45	47.0	95.5	-1.035
46	47.5	93.0 89.0	-4.242
47 48	48.0 48.5	84.0	-8.950 -14.658
49	49.0	77.0	-22.366
50	49.5	70.0	-30.074
51	50.0	117.5	16.719
52	50.5	116.0	14.511
53	51.0	113.5	11.303
54 55	51.5 52.0	110.5 106.0	7.595 2.388
56	52.0 52.5	108.0	3.680

106.0 108.0

3.680

52.0 52.5

56

Table B3. -- continued

VATI	AND	OTHERS.	1982

OBS	AGE	SEALVL	RESID
57	53.0	102.0	-3.028
58	53.5	95.0	-10.736
59	54.0	105.5	-0.943
60	54.5	95.0	-12.151
61	55.O	101.5	-6.359
62	55.5	97.0	-11.567
63	56.0	90.0	-19.274
64	56.5	114.0	4.018
65	57.0	111.5	0.810
66	57.5	107.0	-4.398
67	58.0	97.0	-15.106
68	<b>58</b> .5	79.0	-33.813
69	<b>59</b> .0	98.0	-15.521
70	<b>59</b> .5	92.0	-22.229
71	60.0	89.0	-25.937
72	60.5	133.0	17.356
73	61.0	129.0	12.648
74	61.5	124.5	7.440
75	62.0	119.5	1.732
76	62.5	113.5	-4.975

Table B4.—Frequency (in cycles per one-half mega annum units), period (in one-half mega annum units), periodogram intersity value (P\_O1), and spectral density estimate (S\_O1) (both in arbitrary units) data from Vail and Others, 1982.

VAIL	AND OTHERS,	1982 DATA	WITH WEIGHTS	1 2 3 2 1
OBS	FREQ	PERIOD	P_01	S_01
1	0.00000		0.00	621.772
2	0.08267	76.0000	9007.45	566.608
3	0.16535	38.0000	3634 . 29	448.492
4	0.24802	25.3333	2768.59	366.243
5	0.33069	19.0000	7260.89	331.198
6	0.41337	15.2000	2317.59	271.093
7	0.49604	12.6667	1868.32	203.113
8	0.57871	10.8571	2680.13	130.022
9	0.66139	9.5000	110.25	79.909
10	0.74406	8.4444	390.07	66 . 146
11	0.82673	7.6000	697.99	62.348
12	0.90941	6.9091	, 2014 . 19	74.258
13	0.99208	6.3333	38.68	53.163
14	1.07476	5.8462	492.45	39.054
15	1.15743	5.4286	185.27	23.084
16	1.24010	5.0667	477.40	25.768
17.	1.32278	4.7500	76.58	23.526
18	1.40545	4.4706	465.92	24.694
19	1.48812	4.2222	359.10	19.639
20	1.57080	4.0000	46.25	15.271
21 22	1.65347 1.73614	3.8000 3.6190	42.93	15.274
23	1.81882	3.4545	318.42 510.22	19.157 21.521
23	1.81882	3.4545	58.80	15.856
25	1.98416	3.1667	105.98	12.852
26	2.06684	3.0400	66.04	11.841
27	2.14951	2.9231	375.66	19.031
28	2.23218	2.8148	118.94	21.118
29	2.31486	2.7143	549.38	24.205
30	2.39753	2.6207	115.47	18.962
31	2.48020	2:5333	244.92	15.973
32	2.56288	2.4516	90.60	10.980
33	2.64555	2.3750	110.20	9.545
34	2.72823	2.3030	144.32	7.807
35	2.81090	2.2353	34.18	6.158
36	2.89357	2.1714	70.65	7.524
37	2.97625	2.1111	53.84	9.198
38	3.05892	2.0541	318.63	14.014
39	3.14159	2.0000	66.07	13.974

Table B5.--Age (in mega annum units), paleotemperature, and residuals (in arbitrary units) data from Wolfe, 1978.

PALEOTEMPERATURE	FOR	WOLFE,	1978	DATA

Table B5.--continued

DALFOTEMBERATURE	EOD 1101 EE	4070 DATA
PAI FOTEMPERATURE	FOR WOLFF	1978 DATA

OBS	AGE	SEALVL	RESID
OBS 57890123456678901723456789012345678999999999999999999999999999999999999	AGE 05050505050505050505050505050505050505	SEALVL  205 300 360 381 383 371 360 360 260 208 172 153 143 139 144 156 203 262 317 395 430 440 434 407 290 240 215 202 217 235 265 300 360 360 360 360 360 360 360 360 360	RESID  46.66 139.09 196.52 214.38 199.81 186.25 159.68 121.17 7.14.59 -36.16 -48.73 -55.87 -43.43 1.00 57.43 1.09.86 185.29 217.76 216.59 187.02 67.45 14.88 -30.25 -40.39 -35.52 -40.39 -35.96 -23.52 -8.034 51.77 79.20 96.64 104.07 103.50 98.93 86.36 63.79 31.23 -11.34 -63.91 -114.48 -154.05 -174.61 -181.18 -176.75 -159.32 -1174.61 -181.18 -176.75 -159.32 -110.70 -120.45 -107.02
112	60.5	225	-74.59

Table B5. -- continued

# PALEOTEMPERATURE FOR WOLFE, 1978 DATA

OBS	AGE	SEALVL	RESID
113	61.0	270	-32.158
114	61.5	310	5.273
115	62.0	350	42.705

Table B6.--Frequency (in cycles per one-half mega annum units), period (in one-half mega annum units), periodogram intersity value (P\_O1), and spectral density estimate (S\_O1) (both in arbitrary units) data from Wolfe, 1978.

PALEOTEMPERATURE FOR WOLFE, 1978 DATA WITH WEIGHTS 1 2 3 2 1

obs	FREQ	PERIOD	P_01	s_01
1	0.00000	_	o	10763.2
2	0.05464	115.000	137663	9719.4
3	0.10927	57.500	126824	7995.3
4	0.16391	38.333	19616	5247.0
5	0.21855	28.750	71551	7178.8
6	0.27318	23.000	166	10281.8
7	0.32782	19.167	430861	14659.0
8	0.38245	16.429	137903	11609.0
9	0.43709	14.375	17628	6773.4
10	0.49173	12.778	2099	1680.6
11	0.54636	11.500	2307	452.5
12	0.60100	10.455	5999	415.5
13	0.65564	9.583	10428	429.0
14	0.71027	8.846	1427	278.8
15	0.76491	8.214	73	147.3
16	0.81955	7.667	247	89.9 124.0
17 18	0.87418 0.92882	7.188 6.765	2664 2523	124.0
19	0.98346	6.389	420	87.1
20	1.03809	6.053	95	47.9
21	1.09273	5.750	688	30.9
22	1.14736	5.476	395	24.7
23	1,20200	5.227	37	18.0
24	1.25664	5.000	69	12.0
25	1.31127	4.792	304	11.5
26	1,36591	4.600	71	9.9
27	1.42055	4.423	66	8.2
28	1.47518	4.259	94	6.7
29	1.52982	4 . 107	96	6.7
30	1.58446	3.966	79	6.0
31	1.63909	3.833	60	5.4
32	1.69373	3.710	33	6.4
33	1.74836	3.594	106	8.2
34	1.80300	3.485	207	9.1
35	1.85764	3.382	71	7.1
36 37	1.91227 1.96691	3.286 3.194	15 44	5.8 6.7
38	2.02155	3.194	170	9.1
39	2.07618	3.026	180	9.7
40	2.13082	2.949	61	8.1
41	2.18546	2.875	55	6.4
42	2.24009	2.805	100	5.7
43	2.29473	2.738	57	4.9
44	2.34936	2.674	58	3.8
45	2.40400	2.614	6	2.8
46	2.45864	2.556	33	3.3
47	2.51327	2.500	55	3.8
48	2.56791	2.447	95	4.1
49	2.62255	2.396	3	3.3
50 51	2.67718 2.73182	2.347 2.300	34 56	4.0
52	2.78646	2.300	134	4.6 5.1
53	2.84109	2.212	134	4.0
54	2.89573	2.170	1	3.5
55	2.95037	2.170	84	4.3
			<b>U</b> -1	7.5

Table B6.--continued

# PALEOTEMPERATURE FOR WOLFE, 1978 DATA WITH WEIGHTS 1 2 3 2 1

OBS	FREQ	PERIOO	P_01	S_01
56	3.00500	2.09091	58.075	4.95417
57	3.05964	2.05357	101.892	5.61360
58	3.11427	2.01754	13.787	4.99637

Table B7.--Age (in mega annum units), paleotemperature and residuals (in arbitrary units) data from Poore and Wolfe, 1982.

POORE	AND	WOLFE.	1982	DATA

Table B7. -- continued

POORE AN	D WOLFE	. 1982	DATA
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OBS	AGE	SEALVL	RESID
57	53.0	104.0	17.424
58	53.5	96.0	8.744
59	54.0	90.0	2.064
60	54.5	88.5	-0.116
61	55.0	90.5	1.204
62	55.5	95.5	5.524
63	56.0	101.0	10.343
64	56.5	104.0	12.663
65	57.0	97.5	5.483
66	57.5	82.0	-10.697
67	58.0	59.0	-34.377
68	58.5	50.5	-43.557
69	59.0	52.5	-42.237
70	59.5	65.0	-30.418
71	60.0	77.0	-19.098
72	60.5	86.5	-10.278
73	61.0	95.0	-2.458
74	61.5	103.5	5.362
75	62.0	90.0	-8.818
76	62.5	78.0	-21.498

Table B8.--Frequency (in cycles per one-half mega annum units), period (in one-half mega-annum units), periodogram intensity value (P\_O1), and spectral density estimate (S\_O1) (both in arbitrary units) from Poore and Wolfe, 1982.

POORE AND	WOLF.	1982	DATA	WITH	WEIGHTS	12	32	1
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OBS	FREQ	PERIOO	P_01	S_01
1	0.00000		0.0	457.567
2	0.08267	76.0000	6911.3	493.572
3	0.16535	38.0000	1685.2	471.023
4	0.24802	25.3333	10983.3	496.875
5	0.33069	19.0000	5515.2	403.209
6	0.41337	15.2000	1933.1	283.185
7	0.49604	12.6667	1538.2	165.156
8	0.57871	10.8571	1138.1	117.445
9	0.66139	9.5000	2406.4	107.361
10	0.74406	8.4444	46.0	76.125
11	0.82673	7.6000	1016.6	59.925
12	0.90941	6.9091	487.5	37.442
13	0.99208	6.3333	254.2	27.861
14	1.07476	5.8462	184.5	14.397
15	1.15743	5.4286	27.9	7.767
16	1.24010	5.0667	23.2	5.555
17	1.32278	4.7500	125.3	6.171
18	1.40545	4.4706	67.9	7.134
19	1.48812	4.2222	112.0	7.791
20	1.57080	4.0000	105.5	7.366
21	1.65347	3.8000	73. <b>3</b>	6.845
22	1.73614	3.6190	78.2	6.093
23	1.81882	3.4545	74.9	5.578
24	1.90149	3.3043	52.5	5.363
25	1.98416	3.1667	71.4	5.374
26	2.06684	3.0400	78.3	5.375
27	2.14 <b>9</b> 51	2.9231	57.2	5.244
28	2.23218	2.8148	63.4	4.779
29	2.31486	2.7143	66.8	4.285
30	2.39753	2.6207	24.1	3.569
31	2.48020	2.5333	52.1	3.099
32	2.56288	2.4516	30.2	2.732
33	2.64555	2.3750	18.8	2.626
34	2.72823	2.3030	52.5	2.530
35	2.81090	2.2353	23.2	2.227
36	2.89357	2.1714	14.4	1.866
37	2.97625	2.1111	29.7	1.437
38	3.058 <b>9</b> 2	2.0541	9.6	1.032
39	3.14159	2.0000	2.3	0.925